Oscillation theory of two-term differential equations

Uri Elias

CONTENTS

PREFACE

Oscillation theory was born with Sturm's work in 1836. It has been flourishing for the past fifty years. Nowadays it is a full, self-contained discipline, turning more towards nonlinear and functional differential equations.

Oscillation theory flows along two main streams. The first aims to study properties which are common to all linear differential equations. The other restricts its area of interest to certain families of equations and studies in maximal details phenomena which characterize only those equations. Among them we find third and fourth order equations, self adjoint equations, etc.

Our work belongs to the second type and considers two term linear equations modeled after $y^{(n)} + p(x)y = 0$. More generally, we investigate $L_n y + p(x)y = 0$, where L_n is a disconjugate operator and $p(x)$ has a fixed sign. These equations enjoy a very rich structure and are the natural generalization of the Sturm-Liouville operator. Results about such equations are distributed over hundreds of research papers, many of them are reinvented again and again and the same phenomenon is frequently discussed from various points of view and different definitions of the authors. Our aim is to introduce an order into this plenty and arrange it in a unified and self contained way. The results are readapted and presented in a unified approach. In many cases completely new proofs are given and in no case is the original proof copied verbatim. Many new results are included.

This approach is a subjective one and naturally, it is influenced by the research of the author and reflects his personal taste. As a result, we do not attempt to bring here every single known theorem, but rather represent the development of the mainstream. The effort to achieve a unified theory also caused the omission of some very elegant results from this treatise, since they utilize exclusive or remote methods. For example, we do not pay special attention to low order equations, in spite of the rich literature in the field. Neither do we mention results which are specific to self adjoint equations.

Haifa, 1996 — Uri Elias

Chapter 0

INDEX